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The evolutionary time scale of Sakurai's object: A test of convection theory?

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ABSTRACT

Sakurai's object (V4334 Sgr) is a born again AGB star following a *very late thermal pulse*. So far no stellar evolution models have been able to explain the extremely fast evolution of this star, which has taken it from the pre-white dwarf stage to its current appearance as a giant within only a few years. A very high stellar mass can be ruled out as the cause of the fast evolution. Instead the evolution time scale has now been found in stellar models by making the assumption that the convective efficiency for element mixing in the He-flash convection zone during the very late thermal pulse is smaller than predicted by the mixing-length theory. As a result the main energy generation from fast convective proton capture will occur at a larger mass coordinate, closer to the surface and the expansion to the giant state can be accelerated to a few years, as required by past observations of V4334 Sgr. Assuming a mass of V4334 Sgr of 0.604 $M_\odot$ which is consistent with a distance of 4 kpc, a reduction of the MLT mixing efficiency by a factor of ~ 100 is required to match the evolutionary time scale of V4334 Sgr. This value decreases if V4334 Sgr has a smaller mass and accordingly a smaller distance. However, the effect does not disappear for the smallest possible masses. These findings may present a semi-empirical constraint on the element mixing in convective zones of the stellar interior.

Subject headings: stars: AGB and post-AGB — abundances — evolution — interior — individual: V4334 Sgr

1. Introduction

Sakurai's object (V4334 Sgr) has displayed a dramatically fast evolution both in stellar parameters and in chemical abundance pattern (Duerbeck et al. 1997; Asplund et al. 1999; Duerbeck et al. 2000). In 1976 it was possibly detected by the ESO/SERC survey close to the detection limit of m_J=21 (Pollacco 1999) which coincides with the stellar parameters of a pre-WD in the Hertzsprung-Russell diagram (see Fig. 1). While this measurement is an important consistency check, the last non-detection in 1994 at the limiting magnitude of m_V=15.5 and the first positive detection at m_V=12.4 by Takamizawa (**?**, see)]duerbeck:97 represents a stringent constraint on the evolutionary speed (Fig. 1). This evolution has been interpreted as the result of a final He-flash which occurred in 1994. By early 1996 the star had reached $\log L/\mathrm{L}_\odot \simeq 3.8$ and cooled to well below $\log T_{\rm eff} \sim 4$. Since then it has continued to cool and brighten while displaying RCrBr-like red declines. Thus, according to the observational evidence, V4334 Sgr must have completed the transition (also known as *born-again evolution*) from the pre-white-dwarf stage to its current appearance as a giant in about two years. This interpretation is supported by photoionization modeling of the planetary nebula of V4334 Sgr (Pollacco 1999; Kerber et al. 1999). These models place the central star at an HRD location which is compatible with a pre-WD central star.

Another born-again evolution star is FG Sge which has brightened slowly over the last century (**?**, see)and references therein]vanGenderen:95. The born-again time scale of FG Sge has been successfully used to derive the stellar mass under the assumption that the star has gone through a *late thermal pulse* (Blöcker & Schönberner 1997). Applying the same procedure to V4334 Sgr leads to an excessively large stellar mass of about $1\,\mathrm{M}_\odot$ as noted by Duerbeck et al. (2000). Such a high mass would require the long distance scale ($d \simeq 8\,$kpc) in disagreement with independent distance determinations (**?**, such as the extinction method;)]kimeswenger:98, which yield distances as low as $d \simeq 1.1\,$kpc.

There is more compelling evidence from nucleosynthesis that V4334 Sgr is not very massive. The abundance ratio N/O in the planetary nebula (PN) is well below unity (Pollacco 1999). The PN material reflects the envelope composition during the very last phase on the AGB. According to recent stellar evolution calculations by Lattanzio & Forestini (1999), AGB stars with the highest mass have a continuously increasing N/O ratio due to hot-bottom burning and eventually the ratio exceeds unity, e.g. at $M_{\rm ZAMS} = 6\,\mathrm{M}_\odot$ in the case of solar metallicity. Therefore V4334 Sgr cannot have the highest possible mass.

In fact, V4334 Sgr must be of even lower mass than required by the N/O constraint. Asplund et al. (1999) report a significant lithium abundance which increased over the six month period covered by their observation in 1996 ($0.5 - 1.0$ dex above initial solar). This amount of lithium cannot be inherited from a previous evolutionary phase but must be a nucleosynthesis product of the special conditions of the final flash (Herwig & Langer 2001). Because any mechanism of lithium creation relies on a readily available reservoir of ^{3}He, the progenitor star must have avoided hot-bottom burning altogether and thus V4334 Sgr is less massive than $\sim 0.7\,\mathrm{M}_\odot$.

While observationally the very short evolutionary time scale with a probably normal CSPNe (central star of PNe) seems well established, a theoretical explanation of this phenomenon is lacking. Here it should be noted that two different kinds of models of the final flash have been constructed. Most models are *late thermal pulse* (LTP) evolutionary sequences (which applies to FG Sge). The thermal pulse occurs while the star is still on the horizontal crossing from the AGB to the CSPNe phase at constant luminosity. During this first post-AGB phase the H-shell is still active and prevents the mixing of envelope material into the He-flash convection zone during the thermal pulse. The born-again evolution is energetically driven by the He-flash. For these LTP models $\tau_{\rm BA}$ (= time of return to AGB - time of occurrence of LTP/VLTP) is about $100 - 200$ yr (Blöcker 1995) and thus clearly in disagreement with the observation of V4334 Sgr.

Both the possible ESO/SERC detection in 1976 and the photoionization models indicate that the post-AGB thermal pulse has occurred very late when the star has already been approaching the WD cooling sequence in the Hertzsprung-Russell diagram. In this case of a *very late thermal pulse* (VLTP) the H-burning has stopped and the protons in the envelope are mixed down into the He-flash convection zone where they burn on the convective time scale. Because the simultaneous burning and convective mixing occurs on the same time scale at the same location, a special numerical treatment is required. Iben et al. (1983) circumvented this problem in their model calculation of the VLTP by ignoring the nuclear energy released by proton captures in the He-flash zone. Energetically their model resembles more that of the LTP and they found $\tau_{\rm BA}$ to be of the order of a few hundred years (from their Fig. 1), similar to the LTP born-again evolution. Another VLTP model sequence has been presented by Iben and MacDonald (1995) with $\tau_{\rm BA} = 17$ yr. This is closer to the born-again time scale of V4334 Sgr, although still too large by a factor of $3 - 4$. However, the reason for the difference in the two values of $\tau_{\rm BA}$ is not clear.

The latest VLTP sequence by Herwig et al. (1999) takes the nuclear energy generation by proton captures in the He-flash convection zone into account. For this purpose a numerical scheme has been developed which consistently couples the nuclear network equations with the equations of time-dependent convective element mixing (Herwig & Koesterke 2001). However, for this sequence we found $\tau_{\rm BA} \sim 350$ yr, even somewhat longer than a LTP case derived from the same AGB starting model. Therefore, this model is not in agreement with V4334 Sgr.

In this paper we demonstrate that the very short born again evolution time can be reproduced by stellar models if the convective element mixing efficiency of the He-flash convection zone is assumed to be reduced compared to the mixing velocity predicted by the mixing-length theory (MLT).

2. Results

The born-again evolution of V4334 Sgr is another case of the general problem of why stars become red giants. It is well known that the non-linear stellar structure equations as a boundary value problem have multiple solutions which may be associated with different topologies (**?**, e.g.)]sugimoto:00. If the assumption of thermal equilibrium is relaxed, the transition between solutions of different topologies can be obtained. This leads to the initial value problem of stellar evolution. In order to switch from a dwarf structure to a giant structure the entropy of the envelope has to be increased.

As illustrated in Fig. 4 of Herwig et al. (1999), the peak proton-capture energy release is located deep in the He-flash convection zone in that VLTP model. The entropy increase in these layers by the additional H-burning luminosity barely affects the outermost layers because, in the He-flash convection zone, the temperature is already greatly increased by the ongoing He-flash. If the protons are captured at such a deep position in the intershell region the corresponding energy release is merely a perturbation of the prominent He-shell instability. Then, the ingestion of protons does not significantly change the time scale of the born-again evolution.

The position of peak hydrogen burning is determined by the competing mixing and nuclear time scales. $\tau_{\rm mix}$ is determined according to the assumptions about the efficiency of convective element mixing. In Herwig et al. (1999) we have adopted $D_{\rm MLT} = \frac{1}{3}\alpha_{\rm MLT} H_{\rm p} v_{\rm MLT}$ where $v_{\rm MLT}$ is the convective mixing velocity according to the MLT (Langer et al. 1985). The nuclear time scale $\tau_{\rm nuc}$ decreases with increasing temperature as the reaction rate of proton capture by ^{12}C increases. The main energy generation by fast convective proton burning will occur at that position in the He-flash convection zone where $\tau_{\rm nuc} \simeq \tau_{\rm mix}$. This position moves towards the top of the He-flash convection zone as the convective mixing efficiency is reduced. It can be expected that the evolutionary speed of the born again evolution after a VLTP depends on the position of H-burning energy release within the star.

In order to evaluate the influence of such a change for the born-again evolution new VLTP model sequences have been computed using a starting model of the original sequence of Herwig et al. (1999) before the ingestion of protons into the He-flash convection zone begins. The same evolutionary code (EVOL) has been used. It follows the time evolution of 16 isotopes for hydrogen- and helium-burning. Time-dependent overshoot on any convective boundary can be considered and the latest OPAL opacities have been used (Iglesias & Rogers 1996). The mixing length parameter is $\alpha_{\rm MLT} = 1.7$. All models have been computed with a metallicity of $Z = 0.02$.

We define a new parameter $f_{\rm v} \equiv D_{\rm MLT}/D_{\rm CM}$ where $D_{\rm CM}$ is the diffusion coefficient for composition mixture. VLTP and born again model sequences for $f_{\rm v} = 1$, 3, 30 and 300 have been computed. A $0.535\,{\rm M}_\odot$ post-AGB sequence with a VLTP with a $M_{\rm ZAMS}$=$1\,{\rm M}_\odot$ progenitor model (Herwig et al. 2000) and $f_{\rm v} = 1$ has been computed for comparison.

For larger values of $f_{\rm v}$ the evolution across the HRD is accelerated. The cases $f_{\rm v} = 3$ and $f_{\rm v} = 30$ for the mass $0.604\,{\rm M}_\odot$ are shown in Fig. 1. The AGB is reached within 195 and 9.77 yr for the two cases respectively. In a VLTP model sequence with $f_{\rm v} = 30$ the peak p-capture energy is released at $m_{\rm r} \sim 0.601\,{\rm M}_\odot$ compared to $m_{\rm r} \sim 0.595\,{\rm M}_\odot$ with $f_{\rm v} = 1$.

Models with reduced convective velocity for composition mixing do not only evolve fast, but they also feature a modified evolution of convective zones. In the new computation with the reduced convective mixing efficiency, H-burning takes place in the top layers of the intershell convection zone and establishes its own convective layer on top of the actual He-flash convection zone. In contrast H-burning in the original computation takes place deeper inside the intershell region and the separate H-burning convection zone inside the He-flash convection zone is very short lived.

A comparison of the evolutionary times for different $f_{\rm v}$ values from the last non-detection in 1994 to the first positive pre-discovery detection in 1995 (t2) and from this time to the date of the first spectra reported by Asplund et al. (1999) (t1) with the time interval defined by these dates is given in Fig. 2. If V4334 Sgr has a mass of $0.604\,\mathrm{M}_{\odot}$ then this tentative comparison requires a composition mixing efficiency reduction of $f_{\mathrm{v}} \sim 100$. This follows from the comparson of time interval t1. The fact that time interval t2 requires a larger reduction factor is due to the fact that the time interval t2(SO) is only a lower limit because V4334 Sgr was at or below the detection limit in 1994.

3. Discussion and conclusions

Several tests have been carried out to ensure obvious sources of uncertainty do not jeopardise the general validity of the findings. These, together with more details on the internal evolution and consequences of reduced composition mixing on nucleosynthesis, will be discussed in a forthcoming paper. The main results are the following. Qualitatively, the same results can be obtained when using the old Cox & Stewart (1970) opacities. Overshooting does not effect the result. The influence of numerical parameters such as the mass at which the outer and the interior solutions are attached has been checked. While such parameters do change the results a little and effect the convergence of models, they have not been found to be able to alter the results qualitatively. Note that the convergence is becoming generally worse as the model expands (Fig. 1).

However, several improvements are necessary in future models. We have not considered the μ-barrier during the ingestion of protons which might effect the results to some extend. Moreover, time-dependent treatment of convective energy transport is not considered. This means that the time step has always to be chosen sufficiently larger than the convective mixing time scale according to the MLT in order to be consistent with the assumption of instantaneous convective energy transport. This criterion prohibits the usual time resolution which, for instance, requires that the hydrogen-burning luminosity L_{H} may not increase by more than a few percent. In the VLTP case L_{H} often multiplies by some factor within one time step. However, the comparison of the nuclear energy integrated by the structure equations and the nuclear energy estimated from the amount of consumed proton agreed within 10-20% in all cases. This estimate requires an estimate of the major proton consuming reaction. As f_{v} increases the fraction of $^{13}\mathrm{C}+\mathrm{p}$ reaction increases while for $f_{\mathrm{v}} = 1$ almost all protons are reacting with $^{12}\mathrm{C}$.

The reduced efficiency of composition mixing leads to a different evolution of the convective zones. This will have effects on the CNO element and isotopic ratios. As well, the release of neutrons will be less efficient. In fact, if the $^{13}\mathrm{C}$ formed by p-capture of $^{12}\mathrm{C}$ has no possibility to reach the hotter He-flash convection zone, the neutron source $^{13}\mathrm{C}(\alpha,\mathrm{n})^{16}\mathrm{O}$ will not be activated in the top H-burning convection zone. The time scale for this neutron source at $T_8 \sim 1$ exceeds the short born-again evolution time scale of V4334 Sgr by orders of magnitude.

The formation of lithium is another important test for any stellar model sequence of Sakurai's object. Future detailed studies of the dependence of the nucleosynthesis on the convective mixing efficiency should provide additional constraints on the validity of the proposed concept. From a prelimary analysis of the temperature conditions in models with reduced convective composition mixing we expect that the mechanism of hot hydrogen-deficient ^{3}He-burning (Herwig & Langer 2001) for the synthesis of lithium during the VLTP will provide a lithium abundance in agreement with that of V4334 Sgr.

The computations presented in this paper show that the born again evolution following a VLTP is sensitively dependent on the composition mixing due to convection. Therefore, it is important to consider any mechanism or parameter on which the convective velocity in this region depends. The computations show that the convective velocity in the He-flash convection zone decreases with stellar mass of the post-AGB star. While the $0.604\,\mathrm{M}_\odot$ displays $v_\mathrm{MLT} \sim 3$km/s (at $\Delta m = 0.003\,\mathrm{M}_\odot$ below the top boundary of the He-flash convection zone) the comparison model sequence of mass $0.535\,\mathrm{M}_\odot$ shows only $v_\mathrm{MLT} \sim 0.35$km/s. In accordance with the previous finding the $0.535\,\mathrm{M}_\odot$ model sequence does evolve much faster back to the AGB than the $0.604\,\mathrm{M}_\odot$ sequence (Fig. 1). This faster evolution must be attributed to the lower convective velocity. The evolutionary times for the previously described intervals t1 and t2 are included in Fig. 2 and lie clearly off the relation between f_v and t_BA. However, note that v_MLT of the $0.535\,\mathrm{M}_\odot$ sequence is just about a factor of 10 smaller than in the $0.604\,\mathrm{M}_\odot$ case. Possibly all these models follow a single relation between t_BA and v_CM.

In any case, the assumption of a small mass for V4334 Sgr does not solve the time scale problem alone - for three reasons. First, the time scale for this model sequence is still too slow by an order of magnitude. Second, stellar masses as low as $0.535\,\mathrm{M}_\odot$ are not consistent with the possible detection in the ESO/SERC in 1976. For the $0.535\,\mathrm{M}_\odot$ case the luminosity during the entire post-AGB phase is much higher than the upper limit given by the ESO/SERC measurement. Third, the $0.535\,\mathrm{M}_\odot$ evolutionary track is not consistent with the photoionization models of Pollacco (1999) and Kerber et al. (1999). Instead the $0.604\,\mathrm{M}_\odot$ post-AGB track is in agreement with both the ESO/SERC constraint and the photoionization models. Note, that the horizontal luminosity of the $0.604\,\mathrm{M}_\odot$ post-AGB track is somewhat larger than that of previous computations of comparable mass because of the inclusion of AGB overshooting and the resulting effects on the core-mass luminosity relation described by Herwig et al. (1998). From inspection of Fig. 1 there might be even an upper mass limit from the ESO/SERC constraint. If the mass and accordingly the distance is too large the post-AGB thermal pulse occurs as a *late thermal pulse* without ingestion of protons from the envelope into the He-flash convection zone. In such a case the born again evolution time scale is long, as exhibited by FG Sge.

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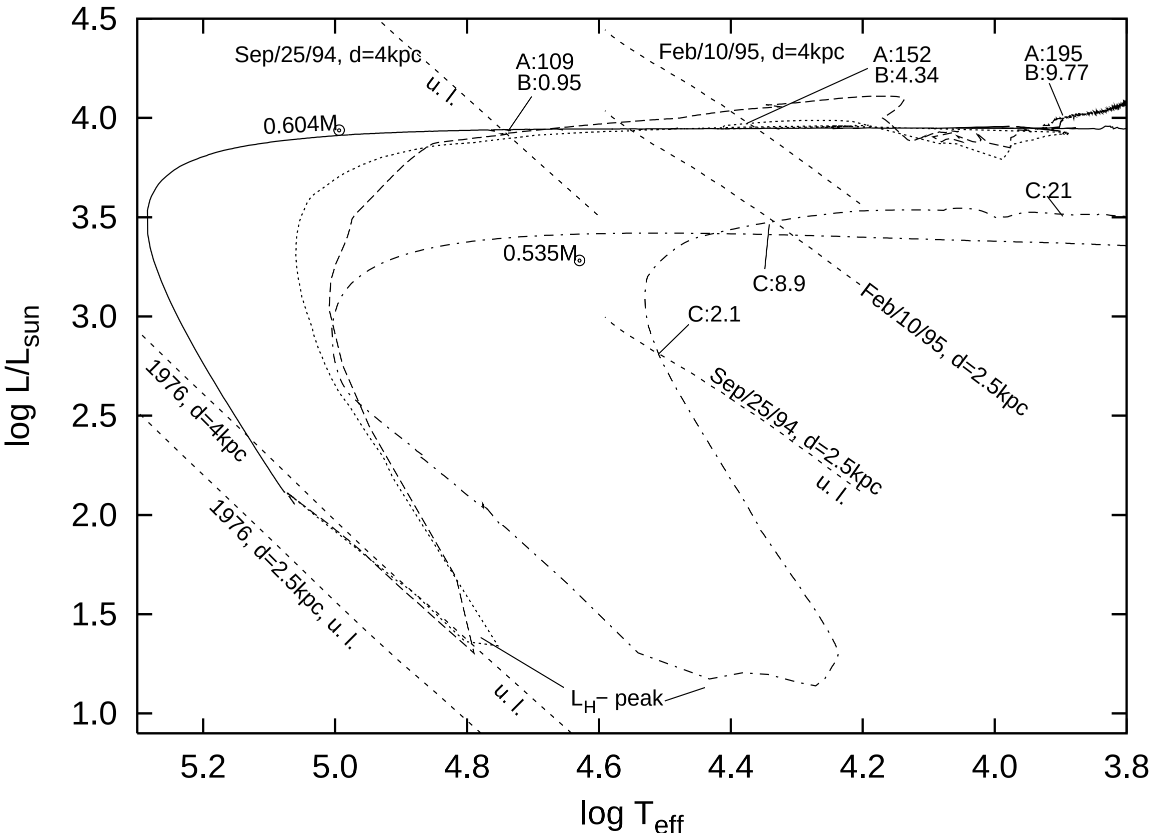

Fig. 1.— Hertzsprung-Russell diagram of the post-AGB sequence of mass $0.604\,\mathrm{M}_\odot$ of Herwig et al. (1999) (solid line) with the VLTP and subsequent born again evolution recomputed with $f_\mathrm{v} = 3$ (dotted line, time labels preceded with A:) and $f_\mathrm{v} = 30$ (long-dashed, time labels preceded with B:); $0.535\,\mathrm{M}_\odot$ post-AGB evolution with VLTP, $f_\mathrm{v} = 1$ (dash-dotted line, time labels preceded with C:). Time labels given at $\log T_\mathrm{eff} = 3.9$ correspond to the first spectra presented in Asplund et al. (1999) which are dated Apr 20, 1996. All time labels are in yr and give the evolutionary time from the moment of largest H-burning luminosity L_H-peak. The dashed straight diagonal lines represent three different magnitudes at or below which V4334 Sgr has been observed at different times (see text). Each magnitude has been plotted for $d = 2.5\,\mathrm{kpc}$ and $d = 4\,\mathrm{kpc}$ for internal consistency with the $0.535\,\mathrm{M}_\odot$ and $0.604\,\mathrm{M}_\odot$ evolutionary track respectively. These distances have been determined from the luminosity-distance relation derived from the data of Duerbeck et al. (2000) for the date of the first spectra found in Asplund et al. (1999) and the corresponding luminosity of the eovlutionary tracks at the temperature determined from the spectra. For the lines of constant magnitudes the following has been used: for $T_\mathrm{eff} > 40000\,\mathrm{K}$ B.C.$_\mathrm{V}$ from Napiwotzki (2001) and for $T_\mathrm{eff} < 40000\,\mathrm{K}$ indices from Bessell et al. (1998) (Kurucz atmosphere models), (V-J)=-0.75, $A_\mathrm{J} = 0.29 A_\mathrm{J}$, $E_\mathrm{B-V} = 0.7$. The lines labeled u. l. are in fact upper limits because V4334 Sgr has been below or close to the detection limit.

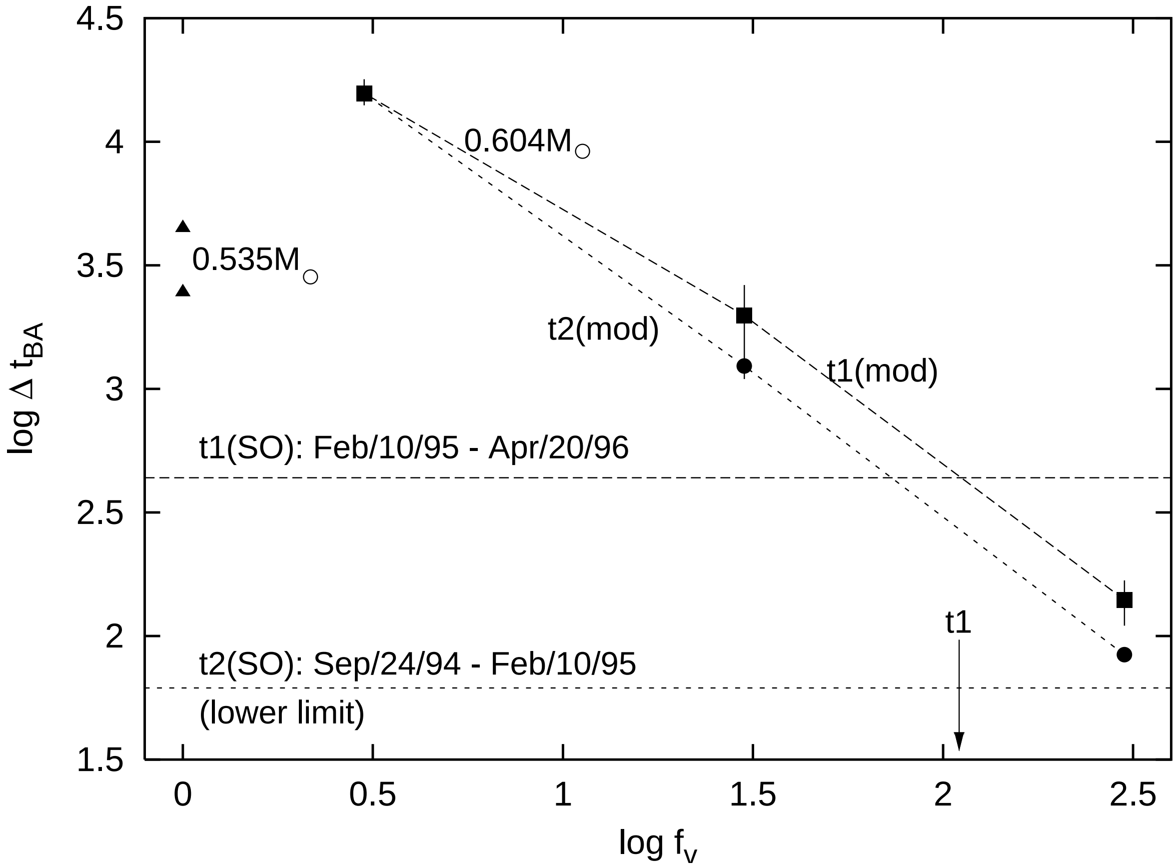

Fig. 2.— Duration $\Delta T_{\rm BA}$ in days of two fractions of the born again evolution of V4334 Sgr (t1(SO) and t2(SO)) and of model predictions as a function of the convective mixing efficiency reduction parameter $f_{\rm v}$. The error bars belong to the evolution interval t1 (squares) from the first positive detection on Feb 10, 1995 to the first spectra analysed by Asplund et al. (1999) and reflect the temperature uncertainty of the latter and an assumed observational error of the magnitude measurement in 1995 of ±0.5mag. The triangulars represent the evolutionary times of the $0.535\,{\rm M}_\odot$ sequence (top triangular belongs to interval t1). The line t2(SO) is labeled lower limit because the corresponding line (Sep/24/94) of constant magnitude in Fig. 1 is an upper limit for V4334 Sgr at that date.